

SOME RECENT DEVELOPMENTS IN RECORD-REPRODUCING SYSTEMS*

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Summary.—Several factors of importance in obtaining satisfactory reproduction of sound from lateral-cut phonograph records are considered. An experimental record-reproducing system employing the principles of frequency modulation is described and data are supplied on the measured and calculated performance characteristics of the system. Curves are included showing the vertical force required for satisfactory tracking with the experimental F.M. pick-up as compared with other pick-ups of conventional design.

The remarkable increase in the sale of phonograph records during the past few years is a definite indication of the returning public interest in records as a medium of home entertainment. If the present interest in phonograph records is to be maintained, on a permanent basis, it is essential that the reproduction of sound from records be at least comparable to or preferably better than that which can be obtained from radio broadcasting.

During the past few years, an investigation was conducted to determine the prospects of materially improving the overall performance of record-reproducing systems. One phase of the investigation was directed toward the possibility of reproducing frequencies up to 10,000 or 12,000 cycles from standard shellac records without the introduction of objectionable surface noise. In the course of this investigation, the possibilities of producing a frequency-modulated signal by means of a special pick-up and associated circuits was investigated. This type of reproducing system appeared to lend itself to a realization of certain requirements which were considered essential to a satisfactory record-reproducing system. It is the purpose of this paper to discuss some of these requirements, to describe the frequency-modulation record-reproducing system, and indicate the improvement in performance which can be expected from such a system.

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(A) PICK-UP REQUIREMENTS FOR SATISFACTORY REPRODUCTION OF LATERAL CUT RECORDS

There are many factors which must be considered in the design of a pick-up to reproduce lateral-cut records. The following factors not only determine the quality of reproduction which will be obtained but also have a direct bearing on the life of the record and stylus.

(1) *Vertical Force Required for Satisfactory Tracking.*—The vertical force required for satisfactory tracking should be low enough to prevent excessive record wear and minimize record surface noise. Numerous tests have been made which indicated that for lacquer records the vertical force should not exceed 20 grams. A maximum value of 30 grams is considered satisfactory for shellac records.

(2) *Mechanical Impedance.*—The vertical and lateral mechanical impedances presented by the pick-up at the stylus should be as low as possible since the work which is performed by the record is a function of these impedances. Low mechanical impedance is likewise desirable to minimize the mechanical noise or chatter radiated directly from the pick-up and record.

(3) *Free Resonance of Pick-Up.*—Experimental evidence indicates that it is desirable to keep the free resonance of the pick-up at as high a frequency as possible to minimize the effect of ticks and other record-groove irregularities.

(4) *Relationship between Stylus Displacement and Audio Output.*—If distortion is to be minimized it is essential that the pick-up and associated circuits provide a linear relationship between audio voltage output and stylus displacement. The necessity for minimizing distortion increases as the frequency range of a sound-reproducing system is extended.

(5) *Frequency Response Characteristic.*—A phonograph pick-up suitable for a high-fidelity system should provide a frequency response throughout the useful audio-frequency range which is proportional to either the amplitude or the velocity of the modulation in the record groove.

(6) *Sensitivity.*—The sensitivity of the pick-up should be such that the amplification required between the pick-up and loud speaker is not so great as to introduce serious microphonic difficulties.

(B) FREQUENCY-MODULATION RECORD-REPRODUCING SYSTEM

A review of the foregoing requirements led to a consideration of several types of record-reproducing systems. The possibility of

producing a frequency-modulated radio signal by means of a special pick-up was investigated. It was found that with a simple pick-up a frequency-modulated signal could be produced which had sufficient frequency deviation to provide a relatively high audio-frequency voltage output, when applied to a frequency discriminator and rectifier combination. Either the inductive or the capacitive reactance of a resonant circuit can be varied to produce a desired frequency shift. From the standpoint of a phonograph pick-up the control of frequency through a variation in capacity seemed to offer the greater advantage.

(1) *F.M. Pick-Up*.—Fig. 1 shows in outline form the general construction of an experimental frequency modulation pick-up. A metal frame or mounting block is provided as a support for an

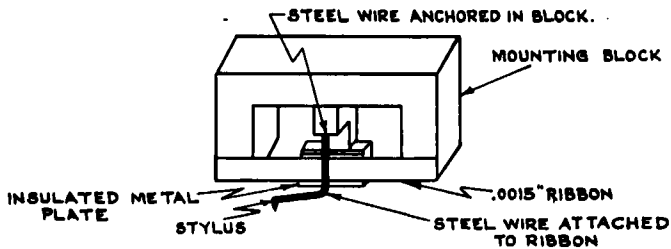


FIG. 1. Experimental frequency modulation pick-up.

insulated plate which is the high-potential side of the pick-up. To this mounting block is also attached a thin metal ribbon. The ribbon, which is mounted in a plane parallel to the insulated plate and spaced from it by a small air-gap, is placed under tension in order to increase the natural resonance frequency of the system. The stylus-supporting wire is anchored to the mounting block at its upper end. It is attached to the ribbon at approximately the mid-point of its length and its free end is bent in a plane essentially parallel to the record groove. The sapphire which is used as a stylus is attached to the end of the wire. The portion of the wire between the ribbon and the sapphire provides sufficient vertical compliance to minimize mechanical noise and to reduce distortion due to pinch-effect. From the figure it is apparent that displacement of the stylus laterally results in a change in the position of the ribbon with respect to the fixed plate and thus produces a change in capacity. The overall

length of the mounting block shown in Fig. 1 is approximately one-half inch. The normal spacing between the fixed plate and the ribbon is approximately 0.004 inch.

From a purely theoretical standpoint it is essential that in a frequency modulation pick-up the change in capacity with displacement of the stylus be such as to produce a linear relationship between frequency change and motion of the stylus. In other words, the variable capacitor formed by the elements of the pick-up should, in radio terminology, be of the straight-line frequency type. From a practical standpoint the distortion introduced by a pick-up constructed along the lines indicated in Fig. 1, when used with circuits to be described later, is sufficiently low as to be substantially negligible.

(2) *Circuit Considerations.*—The essential circuit considerations which are involved in the design of a frequency-modulation record reproducing system may be stated as follows:

- (1) The carrier frequency to be employed.
- (2) A suitable oscillator circuit for use with the pick-up.
- (3) The type of frequency discriminator-rectifier combination to employ.

A study of the question of the operating frequency to use in a frequency modulation phonograph system leads to the conclusion that carrier frequencies as low as those used in the intermediate-frequency amplifiers of radio receivers and as high as those employed for frequency modulation broadcasting will give satisfactory results. If the phonograph is to be used in combination with a radio receiver there may be some advantage in using a carrier frequency which permits the use of one or more of the intermediate-frequency amplifier circuits as a frequency-discriminating network for converting the frequency-modulated signal into amplitude modulation prior to detection. However, if the phonograph is designed as a separate device it may be desirable to use a frequency in the neighborhood of 30 megacycles, particularly in case some frequency in this region is assigned by the F.C.C. for diathermy machines. If a carrier frequency is used in a band thus allocated by the F.C.C. no special shielding would be required to prevent interference with other radio services. The signal level provided at the discriminator by the frequency-modulated oscillator can readily be made quite high, so there is no likelihood of diathermy machines or other electrical equipment causing interference with the phonograph.

Since the oscillator and frequency discriminator-rectifier circuits are to a considerable extent interdependent, they will be discussed together. Fig. 2 is a schematic diagram of circuits which have given very satisfactory results. The circuit problem in connection with the oscillator is to provide an arrangement which will have sufficient frequency stability from the standpoint of line-voltage variations, temperature changes, *etc.*, and at the same time enable the pick-up capacity variations to produce the desired frequency change.

From the standpoint of obtaining the maximum frequency change for a given variation in capacity at the pick-up, it is desirable that the pick-up be connected directly across the oscillator tuned circuit

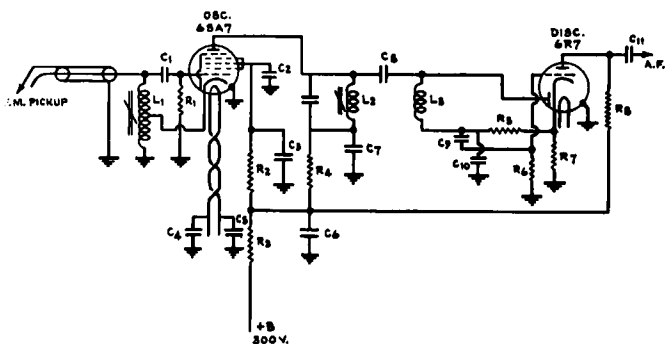


FIG. 2. Oscillator and frequency discriminator-rectifier circuit.

This can, of course, be accomplished by mounting the oscillator tube and associated circuit elements at the pick-up end of the tone arm.

This arrangement has not been found to be particularly desirable because the tone arm is made unduly large and the heat from the oscillator tube causes the end of the tone arm, which is handled by the user, to become uncomfortably hot. The same result, however, can be accomplished by mounting the oscillator tube on the main instrument chassis and connecting it to the pick-up through a resonant transmission line, which is used as the oscillator tuned circuit. It has been found that by connecting the pick-up previously described through a relatively low-capacity line to a conventional oscillator circuit as shown in the diagram a sufficient frequency shift is obtained to give the desired audio-frequency output. In this case the trans-

mission line is treated as a lumped capacity. The line is included as an integral part of the tone arm.

It will be noted that the oscillator tube employed is of the 6SA7 type. This tube permits the use of electronic coupling between the oscillator and discriminator circuits. The oscillator frequency is adjustable by means of an iron core which is associated with the inductance L_1 shown in the diagram.

A simple resonant circuit is utilized as the means for converting the oscillator frequency variations into changes in the amplitude of the signal applied to the diode portion of the 6R7 tube. A powdered-iron core associated with inductance L_2 is used to tune this circuit so that the mean oscillator frequency falls at approximately the 70 per cent response point on one side of the selectivity characteristic. The rectification of the r-f signal by the diode develops an audio-frequency potential across the resistor R_5 . This audio-frequency potential is then amplified by the triode section of the 6R7. The output voltage which appears across resistor R_6 in the plate circuit of the 6R7 is applied to a suitable audio-frequency amplifier and loud speaker.

The audio-frequency output of the circuit shown in Fig. 2 is a function of:

- (1) The oscillator voltage applied to the discriminator.
- (2) The frequency variations in this voltage which are produced by the pick-up.
- (3) The slope of the discriminator network.
- (4) The audio voltage gain obtained from the 6R7.

An experimental pick-up employed in the circuit shown in Fig. 2 has given an rms potential of 6 to 8 volts across resistor R_6 when reproducing a 400-cycle record cut at a groove amplitude of 0.001 inch.

(C) PERFORMANCE CHARACTERISTICS

In the course of development of the frequency modulation pick-up system equations were derived for use in calculating the performance characteristics. Derivations of these equations will be found in the Appendixes. Through the use of these equations the following characteristics were calculated:

- (a) Lateral mechanical impedance.
- (b) Lateral force acting upon stylus.
- (c) Response characteristic of pick-up and tone arm.

- (d) Tracking weight required to overcome vertical force due to lateral velocity.
 (e) Tracking weights and relative outputs to be obtained with different radius styli.

For purposes of comparison measurements were made on an experimental pick-up to determine the last three of these characteristics.

(1) *Calculated Characteristics.*—(a) *Lateral Mechanical Impedance:* If sine-wave motion of the stylus on the record (perfect tracking) is assumed and the tone arm is of rigid construction then

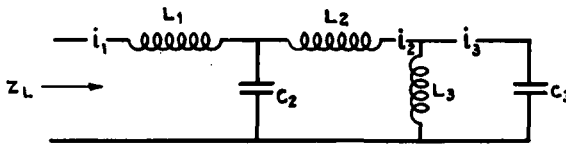


FIG. 3. Equivalent electrical circuit for sine-wave stylus motion.

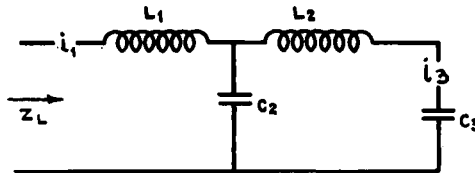


FIG. 4. Equivalent electrical circuit for high frequencies.

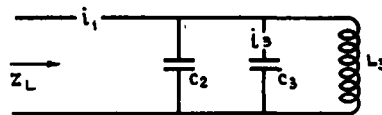


FIG. 5. Equivalent circuit for low frequencies.

the equivalent electrical diagram shown in Fig. 3 may be set up. Since at high frequencies mass L_3 (tone arm) is thousands of times higher than either the stylus or its supporting wire the equivalent diagram shown in Fig. 4 can be used. In this case the lateral impedance Z_{L_1} becomes

$$Z_{L_1} = \left(\frac{j}{\omega} \right) \frac{\omega^4 L_1 L_2 C_2 C_3 - \omega^2 (L_1 C_3 + L_1 C_2 + L_2 C_3) + 1}{\omega^2 L_2 C_2 C_3 - (C_2 + C_3)}$$

Analyzing the above equation, it will be found that the lateral impedance Z_{L_1} becomes infinite at $f = 0$, $f = \infty$ and when $f = f_c$, where

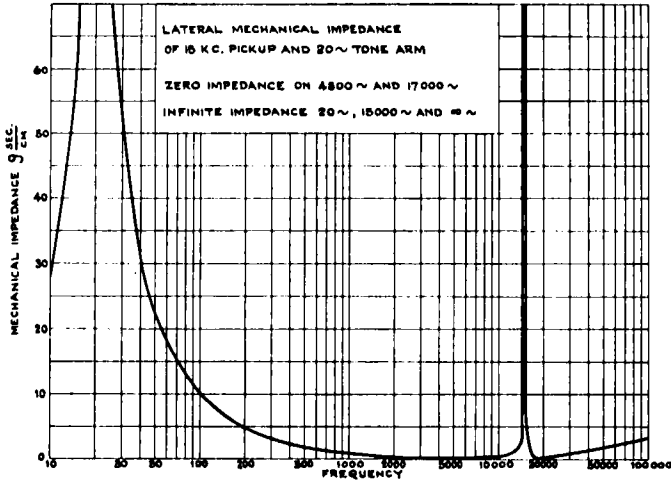


FIG. 6. Calculated impedance of pick-up and tone-arm system.

f_c is the frequency at which the pick-up moving system resonates when the stylus is locked in the groove and may be expressed as

$$f_c = \frac{1}{2\pi \sqrt{L_2 \left(\frac{C_2 C_3}{C_2 + C_3} \right)}}$$

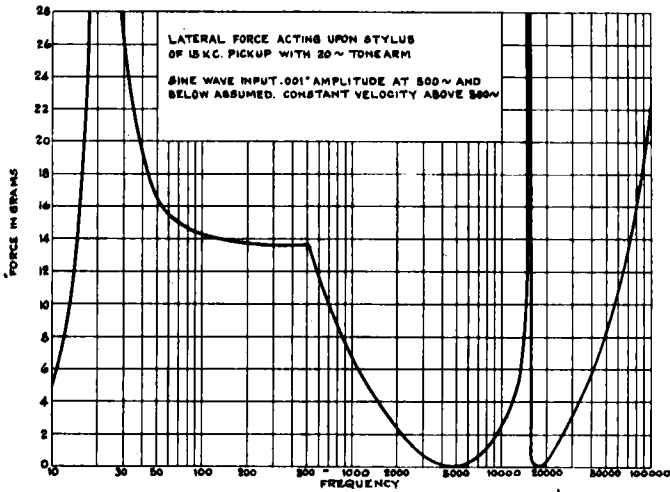


FIG. 7. Lateral force acting upon stylus due to record-groove modulation.

At low frequencies, we may again simplify by disregarding the stylus mass L_1 and the pick-up moving system mass L_2 . Fig. 5 shows the equivalent diagram for low frequencies.

In this case the lateral impedance Z_{L_1} becomes

$$Z_{L_1} = \frac{j\omega L_1}{1 - \omega^2 L_3(C_2 + C_3)}$$

Analyzing this equation it is found that Z_{L_1} becomes infinite at f_s the tone-arm swinging resonance where

$$f_s = \frac{1}{2\pi\sqrt{L_3(C_2 + C_3)}}$$

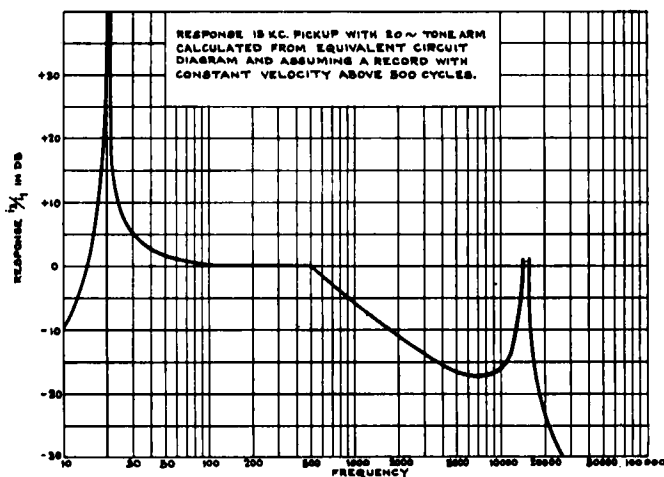


FIG. 8. Calculated response characteristic.

Fig. 6 shows the calculated impedance of the pick-up and tone-arm system based on a pick-up locked resonance of 15 kc and a tone-arm swinging resonance of 20-cycles.

The three points of zero impedance will be noted at $f = 0$, $f = 4800$ -cycles and $f = 17,000$ cycles. Two points of infinite impedance occur as shown at $f = 20$ -cycles and $f = 15,000$ cycles.

(b) *Lateral Force Acting upon Stylus:* Using the calculated values of the effective mechanical impedance of the pick-up and tone-arm system it is possible to calculate the lateral force acting upon the stylus due to record-groove modulation. For these calculations a groove-modulation of 0.001 inch amplitude up to 500-

cycles and a constant groove-velocity above that frequency are assumed. Fig. 7 shows the curve obtained on the basis of these assumptions. It will be noted that the lateral force exerted on the stylus reaches a theoretically infinite value at 20-cycles and at 15,000-cycles, the resonance frequencies of the tone-arm and pick-up.

(c) *Response Characteristic of Pick-Up and Tone Arm:* Making use of the equivalent diagrams in Figs. 4 and 5 the overall response characteristic of the system can be calculated. From these figures it can be seen that current i_3 represents the velocity of the ribbon with respect to the insulated plate and current i_1 represents the velocity of the stylus. The ratio of i_3 to i_1 will provide the response

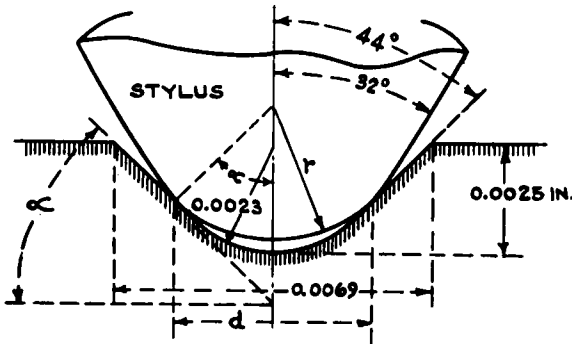


FIG. 9. Stylus seated in record groove.

characteristic of the pick-up with respect to frequency. For high-frequencies this can be calculated from the following formula

$$\frac{i_3}{i_1} = \left(\frac{C_3}{C_2 + C_3} \right) \times \left(\frac{1}{1 - (2\pi f)^2 L_2 \frac{C_2 C_3}{C_2 + C_3}} \right)$$

For low frequencies, the response characteristic may be obtained from

$$\frac{i_3}{i_1} = \left(\frac{C_3}{C_2 + C_3} \right) \times \left(\frac{1}{1 - (2\pi f)^2 L_3 (C_2 + C_3)} \right)$$

It will be noted that two peaks in response occur, one at tone-arm resonance and one at the high-frequency resonance of the pick-up moving system. Fig. 8 shows the response characteristic as calculated from the above equations.

(d) *Tracking Weight Required to Overcome Vertical Force Due to Lateral Velocity:* For proper tracking the stylus must have sufficient vertical force exerted upon it to overcome the vertical component of force due to the lateral velocity of the modulated record groove. Calculations have been made which show the vertical forces exerted upon styli of various radii when seated in a standard groove having

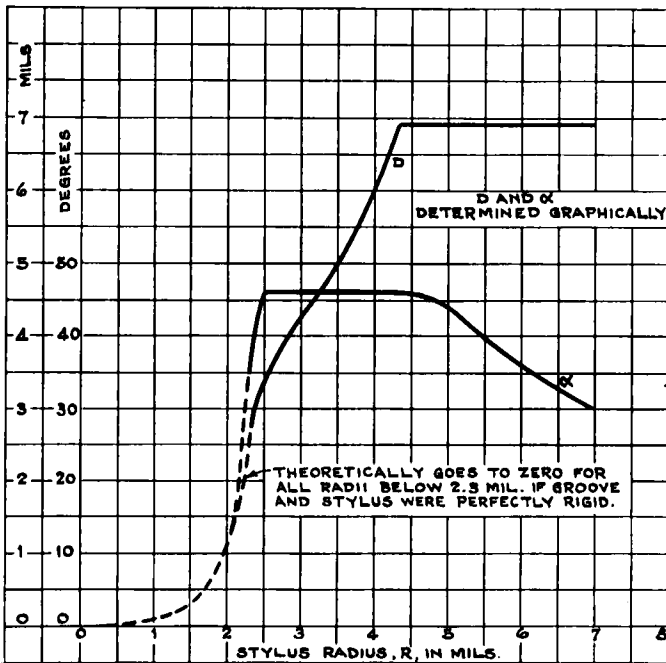


FIG. 10. Variation of d and α with stylus radius.

an 88-degree included angle, a 0.0023-inch radius cutting stylus and a groove width at the top of 0.0069 inch. In addition to the vertical forces, consideration has also been given to the variations to be expected in pinch-effect with different sizes of reproducing styli.

Fig. 9 illustrates a stylus seated in a record groove of the above dimensions. Two important factors which change the diameter of the stylus are: the tracking diameter (d) and the wedging-angle (α). Tracking diameter d has a direct bearing upon both pinch-effect and the high-frequency response and should be kept as small

as possible. On the other hand, the wedging-angle α , which determines the tendency of the stylus to climb the groove wall should be made as large as possible for a specified groove. From this it is obvious that a compromise must be made. Fig. 10 shows the varia-

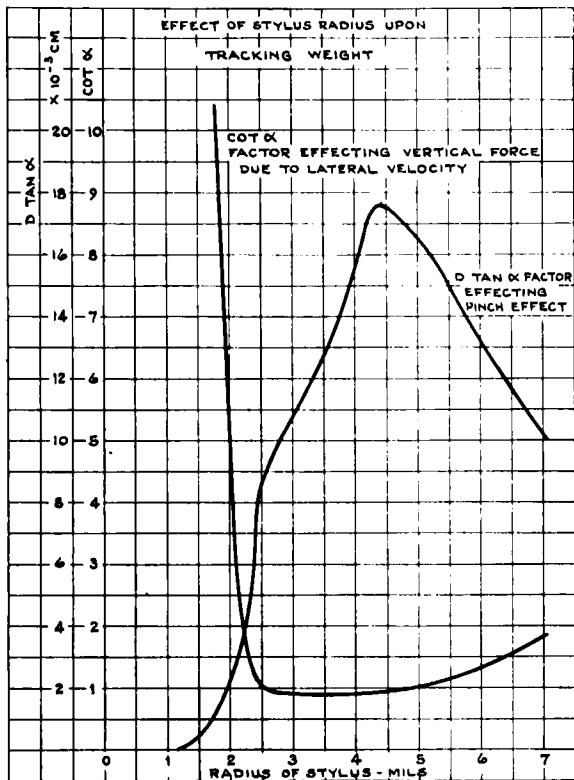


Fig. 11. Effect of stylus radius upon tracking weight.

tions in d and α with stylus radius, and from observation it can be seen that the stylus radius should not be less than 0.0025 inch or greater than 0.0042 inch. Furthermore, since the curve for angle α is flat from a stylus radius of 0.0025 inch to 0.0042 inch and the curve for diameter d is rising rapidly over this range it appears desirable, when record groove variations are considered, to use a stylus radius of about 0.003 inch. The importance of the tracking-

diameter d and the wedging-angle α is further emphasized by the curves in Fig. 11. This figure shows, in curve form, the two factors $\cot \alpha$ and $d \tan \alpha$ which have a direct bearing upon the vertical force due to lateral velocity and pinch effect, respectively. It will be observed that the curve for $\cot \alpha$ approaches infinity for a stylus smaller than 0.0023 inch radius, decreases rapidly to about 0.0025 inch radius and then remains essentially flat to 0.0042 inch. At the same time, however, $d \tan \alpha$, the factor governing pinch-effect, increases rapidly from a 0.0023-inch radius to a 0.0042-inch radius and then decreases. Observation of these two curves provides a further confirmation that a stylus radius of 0.003 inch is the best compromise from the standpoint of overall performance.

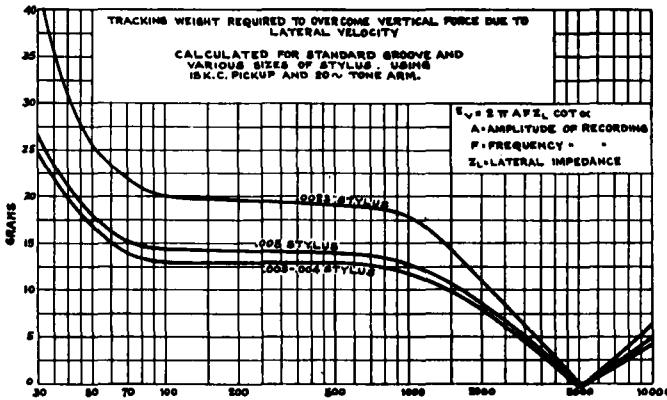


FIG. 12. Calculated tracking weights for styli of different radii.

In the above equations and those which follow such factors as the elasticity of record materials and the effective damping of the mechanical system are not included. For this reason the calculated performance characteristics do not provide all the information that might be desired but they do give a general indication of the performance which can be expected. Expressed mathematically, the conditions for tracking a laterally cut record exist when

$$E_V = Z_L i_L \cot \alpha$$

where E_V = minimum vertical force acting on the stylus which will insure proper tracking.

Z_L = impedance of stylus and pick-up moving system in a lateral direction.

i_L = velocity of recording.

α = angle at which stylus would ride up groove wall.

Utilizing the above equation and the previously determined values for the lateral impedance and the curve for angle α , it is possible to calculate the tracking-weight required to overcome the vertical force due to lateral velocity as the stylus radius is varied. Fig. 12 illustrates in curve form the calculated tracking-weights for stylus radii of 0.0023 inch, 0.003 inch, 0.004 inch, and 0.005 inch. It will be noted that a stylus radius of 0.003 inch to 0.004 inch required the least tracking-weight.

(e) *Tracking-Weights Required and Relative High-Frequency Outputs to Be Obtained with Different Radius Styli:* As the stylus radius

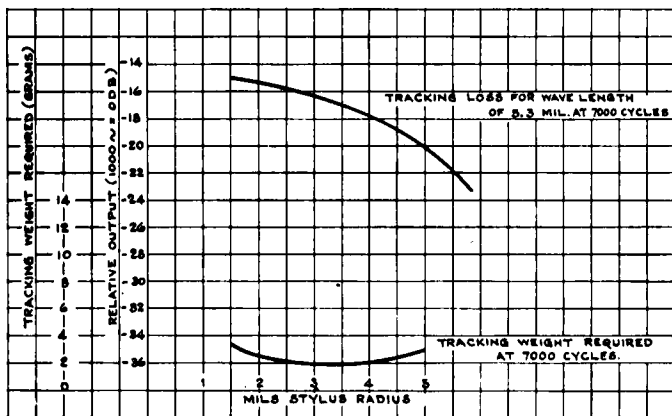


FIG. 13. Tracking loss at 7000 cps for styli of various radii.

is increased, it is apparent that for a given groove velocity the output to be obtained at high frequencies will decrease. Calculations have been made of the expected loss at high frequencies, and Fig. 13 shows the curve for this tracking loss at 7000 cycles for styli from 0.0015 inch radius to 0.005 inch radius. It will again be noted that a stylus radius of 0.003 inch is indicated.

(2) *Measured Characteristics—(a) Response Characteristics of Pick-Up and Tone Arm:* Fig. 14, curve A shows the overall response characteristic of the pick-up, tone arm, and discriminator as obtained from a frequency record having a 500-cycle crossover point between constant amplitude and constant velocity. The rounded portion of this curve at the crossover frequency is due to the limitations imposed by the electrical network used to provide the recording

characteristic. For the purpose of comparison the calculated response characteristic previously shown in Fig. 8 is included in this figure as curve B.

(b) *Tracking-Weight Required to Overcome Vertical Force Due to*

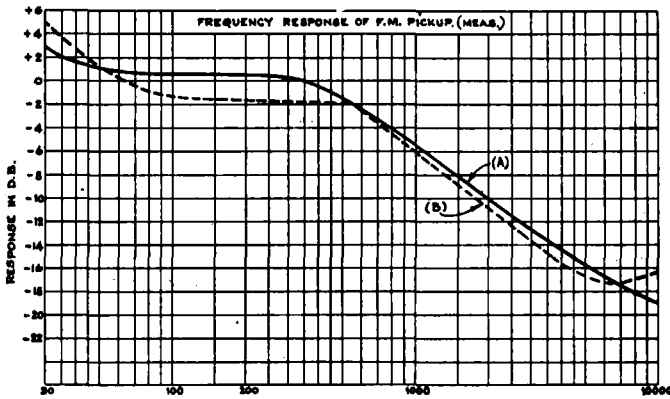


FIG. 14. Frequency response of F.M. pick-up.

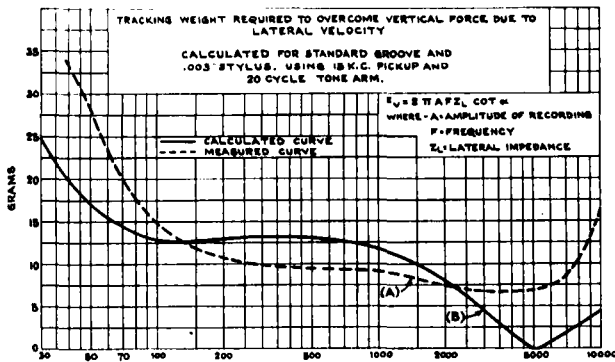


FIG. 15. Measured and calculated minimum weights for satisfactory tracking with experimental pick-up.

Lateral Velocity.—A curve of the measured minimum weight required for satisfactory tracking with an experimental pick-up is shown in Fig. 15 as curve A. Curve B in this figure gives the calculated values for the same radius of stylus. The discrepancy between the curves at the higher frequencies is due to the fact that such factors

as pinch effect, elasticity of the record material, and the Q of the mechanical system were not included in the calculations.

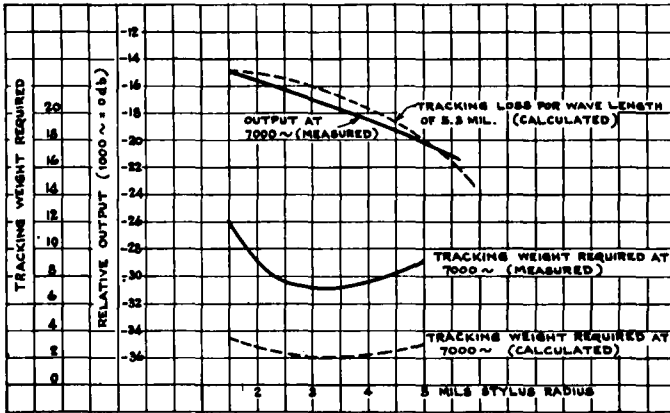


FIG. 16. Correlation between calculated and measured loss in response with increase of stylus radius.

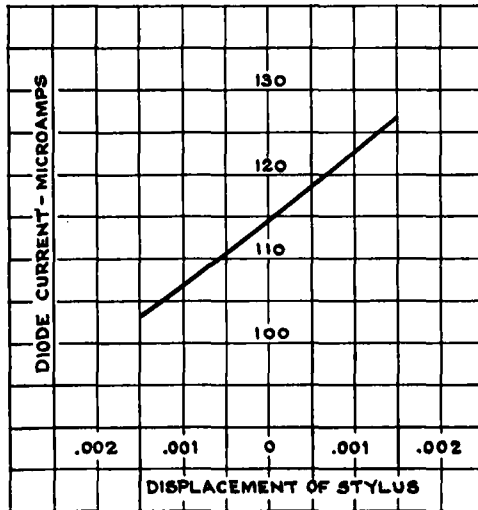


FIG. 17. Relation between diode current and stylus displacement.

(c) *Tracking-Weights Required and Relative High-Frequency Outputs to Be Obtained with Different Radius Styli:* Fig. 16 shows the correlation between calculated and measured loss in response at

7000 cycles as the stylus radius is increased. Also shown in the same figure are calculated and measured curves of the tracking-weight required for different styli at 7000 cycles. It will be noted that while the actual weights required are somewhat higher than the calculated values, because the factors previously mentioned were not included in the calculations, the curve shapes are similar indicating that the choice of a 0.003-inch stylus is a desirable compromise.

(d) *Change in Diode-Current with Stylus Displacement:* Fig. 17 shows the overall linearity existing between current in the diode resistor and displacement of the stylus. This curve shows the combined effect on linearity of the following factors:

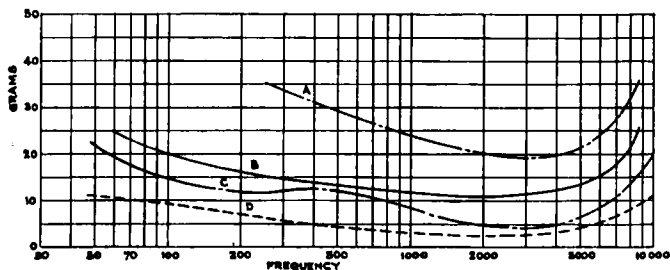


FIG. 18. Tracking weight required for F.M. pick-up and three other types of phonograph pick-ups.

- (1) Change in the capacity of the pick-up with displacement of the stylus.
- (2) Change in frequency of the oscillator with change in capacity of the pick-up.
- (3) Change in output of the frequency discriminator-rectifier combination with change in frequency.

The departure from linearity of this curve represents a distortion of approximately 2 per cent second harmonic and 0.1 per cent third harmonic. The change in frequency which corresponds to the ± 0.0015 -inch displacement of the stylus is $\approx 15,000$ cycles.

(e) *Comparison of Tracking-Weights Required for Various Pick-Ups:* Fig. 18 shows the tracking-weight required for a frequency modulation pick-up as compared with three other types of phonograph pick-ups. Curve A is a conventional crystal pick-up having a normal tracking-weight of 70 grams. Curve B is a transcription-type magnetic pick-up with a normal tracking weight of 45 grams. Curve C shows the tracking-weight characteristic of a recently developed crystal pick-up which operates at a normal tracking-weight

of 28 grams, and curve *D* shows the tracking-weight characteristic of the frequency modulation pick-up normally operated with a tracking-weight of 18 grams.

Conclusions.—An experimental frequency-modulation record-reproducing system, of the type described, has been in use for some time. All the evidence to date indicates that the system is a practical one and is not adversely affected by changes in temperature, humidity, or line voltage.

The experimental frequency modulation pick-up meets the requirements of a satisfactory pick-up to a degree which has not previously been attained in a relatively inexpensive device. The general performance characteristics of a pick-up of this type can be calculated within reasonable limits.

From the listener's standpoint, the experimental frequency modulation phonograph system which has been described makes it possible when using conventional shellac records to extend the frequency range of a record-reproducing system to 10,000 or 12,000 cycles with surprising freedom from surface noise, mechanical noise, and distortion. Further reduction in surface noise can be obtained with shellac records if they are recorded with a high-frequency accentuation characteristic which is comparable to that used in transcriptions. Experimental records of this type have been made. The surface noise obtained from these records with the frequency-modulation reproducing system was reduced to the point where it was not objectional to the most critical listeners.

Although the calculations and measurements which have been given are confined primarily to 78-rpm records, the same performance advantages are retained in a frequency-modulation reproducing system designed for transcriptions.

The writers wish to acknowledge the valuable assistance of Messrs. H. Belar and R. Snepvangers during the development of the frequency-modulation record-reproducing system.

APPENDIX A

Lateral Mechanical Impedance

The equivalent diagram of the frequency modulation pick-up with a rigid tone-arm can be shown as indicated in Fig. 3, where

L_1 = effective mass of needle point.

L_2 = effective mass of ribbon and needle referred to point.

L_3 = inertia of tone arm and pick-up referred to point.

C_2 = compliance of needle between point and ribbon.

C_3 = compliance of ribbon and needle referred to point.

\dot{i}_1 = velocity of needle point.

\dot{i}_3 = velocity of ribbon with respect to support.

Z_L = mechanical impedance in lateral direction.

In practice the mass of the tone arm is several hundred thousand times larger than that of either the point or needle. In view of this, further simplification is possible and at high frequencies the equivalent diagram can be set up as shown in Fig. 4.

For these frequencies the lateral mechanical impedance becomes:

$$\begin{aligned} Z_{L_1} &= \rho L_1 + \frac{\frac{1}{\rho C_2} \left(\rho L_2 + \frac{1}{\rho C_3} \right)}{\frac{1}{\rho C_2} + \rho L_2 + \frac{1}{\rho C_3}} \quad \text{where } \rho = j\omega \\ &= \frac{\frac{L_1}{C_2} + \rho^2 L_1 L_2 + \frac{L_1}{C_3} + \frac{L_2}{C_2} + \frac{1}{\rho^2 C_2 C_3}}{\frac{1}{\rho C_2} + \rho L_2 + \frac{1}{\rho C_3}} \\ &= j \frac{-\omega^2 L_1 L_2 + \omega \left(\frac{L_1}{C_2} + \frac{L_1}{C_3} + \frac{L_2}{C_2} \right) - \frac{1}{\omega C_2 C_3}}{\frac{1}{C_2} + \frac{1}{C_3} - \omega^2 L_2} \\ &= \left(\frac{j}{\omega} \right) \frac{\omega^2 L_1 L_2 C_2 C_3 - \omega^2 (L_1 C_3 + L_1 C_2 + L_2 C_3) + 1}{\omega^2 L_2 C_2 C_3 - (C_2 + C_3)} \end{aligned}$$

From the above formula it can be seen that infinite impedance occurs when $f = \infty$; $f = 0$ or when

$$\omega = \sqrt{\frac{1}{L_2 \frac{C_2 C_3}{C_2 + C_3}}} \quad \text{or} \quad f = \frac{1}{2\pi \sqrt{L_2 \frac{C_2 C_3}{C_2 + C_3}}}$$

At low frequencies L_1 and L_2 become so small in value that they may be neglected and the equivalent diagram may be as shown in Fig. 5.

The mechanical impedance of the pick-up and tone-arm system at low frequencies may be written as:

$$\begin{aligned} Z_{L_2} &= \frac{\frac{1}{\rho(C_2 + C_3)} \times \rho L_3}{\frac{1}{\rho(C_2 + C_3)} + \rho L_3} = \frac{\rho L_3}{1 + \rho^2 L_3 (C_2 + C_3)} \\ &= \frac{j\omega L_3}{1 - \omega^2 L_3 (C_2 + C_3)} \end{aligned}$$

Infinite impedance will then result when

$$\omega = \sqrt{\frac{1}{L_3 (C_2 + C_3)}}$$

or when

$$= \frac{1}{2\pi \sqrt{L_3 (C_2 + C_3)}}$$

This point of infinite impedance occurs when the mass of the tone-arm resonates with the total compliance of the pick-up, and has been generally given the name of "swinging resonance."

APPENDIX B

Response Characteristics

Assuming sine-wave motion of the pick-up point (perfect tracking) and referring to the equivalent diagram, Fig. 4, it can be seen that the frequency response characteristic may be calculated from the ratio i_3/i_1 , for high frequencies.

Since $i_1 = i_2 + i_3$ and also

$$\frac{i_2}{\rho C_2} = i_3 \left(\rho L_2 + \frac{1}{\rho C_3} \right) \text{ then } i_2 = i_3 \left(\rho^2 L_2 C_2 + \frac{C_2}{C_3} \right)$$

Substituting we obtain

$$i_1 = i_3 + i_3 \left(\rho^2 L_2 C_2 + \frac{C_2}{C_3} \right)$$

the ratio i_3/i_1 becomes

$$\begin{aligned} \frac{i_3}{i_1} &= \frac{1}{1 + \frac{C_2}{C_3} + \rho^2 L_2 C_2} = \frac{C_3}{(C_2 + C_3) \left(1 - \omega^2 L_2 \frac{C_2 C_3}{C_2 + C_3} \right)} \\ &= \left(\frac{C_3}{C_2 + C_3} \right) \left(\frac{1}{1 - (2\pi f)^2 L_2 \frac{C_2 C_3}{C_2 + C_3}} \right) \end{aligned}$$

Since the ratio of i_3/i_1 becomes infinite when

$$\omega = \sqrt{\frac{1}{L_2 \frac{C_2 C_3}{C_2 + C_3}}}$$

the frequency of infinite response becomes

$$f_c = \frac{1}{2\pi \sqrt{L_2 \frac{C_2 C_3}{C_2 + C_3}}}$$

By similar reasoning and making use of the equivalent diagram, Fig. 5, the low-frequency response at resonance may be calculated. In this case,

$$\frac{i_3}{i_1} = \frac{C_3}{(C_2 + C_3) (1 - \omega^2 L_3 (C_2 + C_3))} = \left(\frac{C_3}{C_2 + C_3} \right) \left(\frac{1}{1 - (2\pi f)^2 L_3 (C_2 + C_3)} \right)$$

Here again the ratio of i_3/i_1 becomes infinite when

$$\omega = \sqrt{\frac{1}{L_3 (C_2 + C_3)}}$$

and the frequency of infinite response becomes

$$f_L = \frac{1}{2\pi \sqrt{L_3 (C_2 + C_3)}}$$