# Color Volume and Hue-Preservation in HDR Tone Mapping

By Adam Burke, Michael D. Smith, and Michael Zink


## Abstract

*The electrical–electrical transfer function (EETF), described in International Telecommunications Union—Radiocommunication (ITU-R) BT.2390-7, offers four variations, which allow high luminance imagery to be tone-mapped to a lower luminance in different ways. This is useful when converting high dynamic range (HDR) encoding spaces or rendering HDR content on screens that have limited performance. However, for some input colors, these variations introduce drastic changes in hue, in some cases, and do not restrict the output to the desired color volume, in other cases. In this article, a fifth variation is proposed, that preserves the input color's hue while also restricting the output to the specified color volume. This proposed variation is compared and shown to outperform the four standardized variations.*



## Keywords

*Electrical–electrical transfer function (EETF), high dynamic range (HDR), hybrid log-gamma (HLG), International Telecommunications Union—Radiocommunication (ITU-R) BT.2390, maxRGB, perceptual quantizer (PQ), tone mapping*


## Introduction

The electrical–electrical transfer function (EETF), described in International Telecommunications Union—Radiocommunication (ITU-R) BT.2390-7,[1] can be used to map code values representing luminance, from a starting peak luminance to a new, lower luminance. This can be used when mapping an image with a peak of 4,000 cd/m$^2$ to a peak of 1,000 cd/m$^2$, while preserving the colors' hue and creative intent as much as possible. These changes in luminance may be necessary in a variety of cases, such as when converting between the perceptual quantizer (PQ) and hybrid log-gamma (HLG), which support the luminance values of 10,000 and 1,000 cd/m$^2$, respectively. While HLG is indeed a relative-luminance system, the reference condition described in section 7.2 of ITU-R BT.2390-7 for PQ to HLG conversion is the 1,000 cd/m$^2$ operating point. In addition, tone mapping can be implemented within a display that is not capable of rendering the encoded luminance. **Figure 1** illustrates the change in hue when clipping is implemented in place of more advanced tone mapping.

**The electrical–electrical transfer function (EETF), described in International Telecommunications Union—Radiocommunication (ITU-R) BT.2390-7,1 can be used to map code values representing luminance, from a starting peak luminance to a new, lower luminance. This can be used when mapping an image with a peak of 4,000 cd/m$^2$ to a peak of 1,000 cd/m$^2$, while preserving the colors' hue and creative intent as much as possible. These changes in luminance may be necessary in a variety of cases, such as when converting between the perceptual quantizer (PQ) and hybrid log-gamma (HLG), which support the luminance values of 10,000 and 1,000 cd/m$^2$, respectively.**

## Goals

When converting imagery from PQ to HLG, tone mapping should be implemented when PQ code values exceed 1,000 cd/m$^2$. This ensures that colors outside the 1,000 cd/m$^2$ color volume are not clipped, and each color's hue is preserved. It is important that the tone-mapping algorithm accomplishes two objectives: (1) it restricts the color volume of the output image to 1,000 cd/m$^2$ and (2) it preserves the hue angle of each color. The latter can be quantified using the absolute difference in the hue angle between the mapped color and the original color. Meeting these objectives should help ensure that creative intent will be preserved when converting between the encoding spaces.

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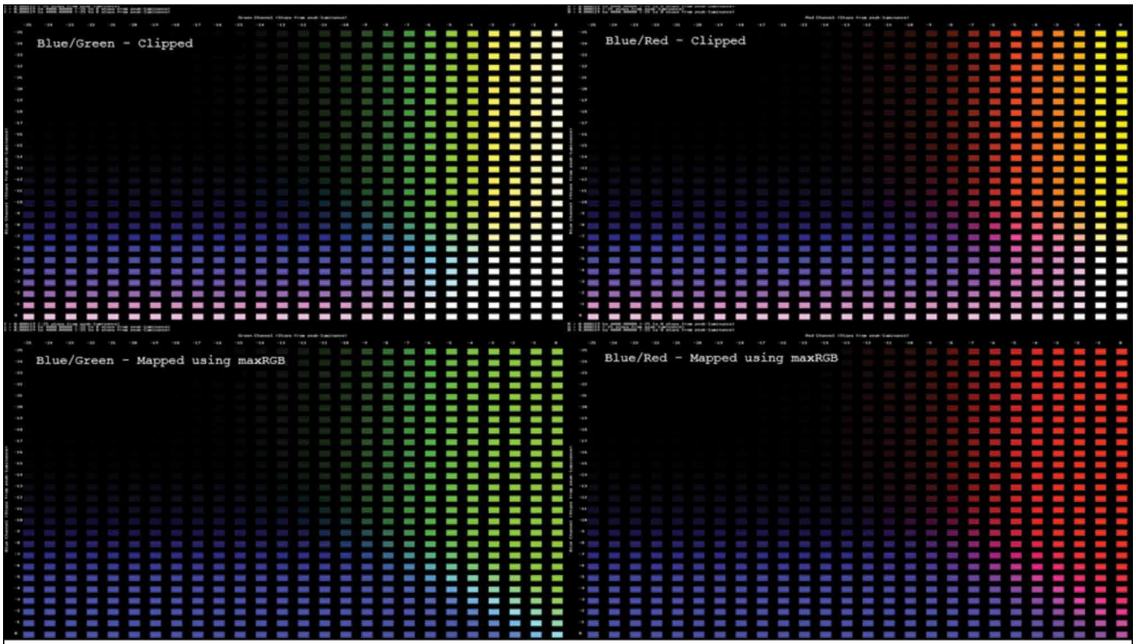


Images are representative of color differences. Colors may vary based on the printing or display gamut.

**FIGURE 1.** Illustration of color conversion utilizing tone mapping (bottom) compared to clipping (top) to go from 4,000 cd/m$^2$ to 1,000 cd/m$^2$.

## Variations Within the Standard

As of BT.2390-7, four implementations of the EETF function are presented with the mapping performed in various color spaces.[1]

These variations have been reproduced below.

**1. $IC_TC_P$**

$$I_2 = \mathrm{EETF}(I_1) \quad (1)$$

$$C_{T2}, C_{P2} = \min\left(\frac{I_1}{I_2}, \frac{I_2}{I_1}\right) \times (C_{T1}, C_{P1}). \quad (2)$$

**2. $Y'C'_BC'_R$**

$$Y'_2 = \mathrm{EETF}(Y'_1) \quad (3)$$

$$C'_{B2}, C'_{R2} = \min\left(\frac{Y'_1}{Y'_2}, \frac{Y'_2}{Y'_1}\right) \times (C'_{B1}, C'_{R1}). \quad (4)$$

**3. YRGB**

$$Y_1 = 0.2627\,R_1 + 0.6780\,G_1 + 0.0593\,B_1 \quad (5)$$

$$Y_2 = \mathrm{EOTF}_{PQ}\left(\mathrm{EETF}\left(\mathrm{EOTF}_{PQ}^{-1}(Y_1)\right)\right) \quad (6)$$

$$(R_2, G_2, B_2) = \frac{Y_2}{Y_1} \times (R_1, G_1, B_1). \quad (7)$$

**4. R'G'B'**

$$(R'_2, G'_2, B'_2) = \mathrm{EETF}(R'_1, G'_1, B'_1). \quad (8)$$

The EETF of 4,000–1,000 cd/m$^2$ is plotted in **Fig. 2**.

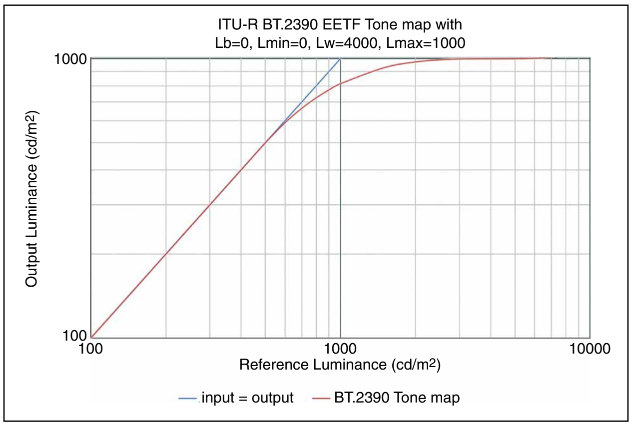


**FIGURE 2.** ITU-R BT.2390 EETF tone map.

## Evaluation Process

The 4,000 cd/m$^2$ equivalent, nonlinear PQ color patches of P3D65[2], red, green, and blue (R', G', B') had, each variation of the EETF function applied to them, reducing the luminance to 1,000 cd/m$^2$. When applicable, the code values were converted to the respective color/encoding space, where the EETF was applied and the resulting code values were converted back to R'G'B'. These code values were recorded as nonlinear red, green, and blue values (R'G'B'). The PQ electro-optical transfer function was then applied to each of these values, generating linear luminance values for each channel (RGB). These values were then recorded.

### *Notes*

- R, G, B values of P3D65 were encoded using the ITU-R BT.2020 primaries.
- Although 1,000 cd/m$^2$ is used throughout this article as an example, in practice, the target output luminance is a variable parameter of the EETF equations and can be adjusted to fit each implementation's requirements.

| TABLE 1. Standard EETF variations and objectives. | | | | |
|---|---|---|---|---|
| | $IC_TC_P$ | $Y'C'_BC'_R$ | YRGB | R'G'B' |
| Restricts To 1,000 cd/m² Color Volume | ✗ | ✗ | ✗ | ✓ |
| Minimizes Difference in Hue | ✓ | ✓ | ✓ | ✗ |

## Limitations of Each Variation

The R'G'B' color space EETF (Eq. 8) does constrain the data within the specified luminance values of the RGB cube. However, this method generates a significant error when mapping the colors. This error can be attributed to each of the channels being processed independently from one another. This is highly noticeable when viewing the images after processing. Calculations and plots showing this error can be found in the section titled "Implemented Results."

Converting to an alternative color space and leveraging another variation (Eqs. 1–7) generates values outside the specified max-luminance when converted back to RGB. These methods thus generate colors outside the 1,000 cd/m² RGB color volume used in this evaluation process. These variations do not constrict the luminance to 1,000 cd/m² as they all work from a weighted luminance or luma channel. The limitations of these variations are summarized in **Table 1**. For example, when converting to YRGB (Eqs. 5–7), the luminance channel is not weighted uniformly. The EETF is then applied to these values, which do not accurately represent the RGB code values.

Example, using YRGB (Eqs. 5–7):

$$R_1, G_1, B_1 = [0, 0, 4000]\ \mathrm{cd/m^2} \rightarrow Y_1 = 237.2$$
$$\mathrm{EOTF}_{PQ}^{-1}(Y_1) = 0.597021 \rightarrow \mathrm{EETF}$$
$$(0.597021) = 0.597021$$
$$Y_2 = \mathrm{EOTF}_{PQ}(0.597021) = 237.198$$
$$\therefore Y_1 \approx Y_2$$
$$B_2 = \frac{Y_2}{Y_1} \times B_1 \approx 4000\ \mathrm{cd/m^2} \blacksquare$$

Additional calculation results for each variation can be found in the section titled "Implemented Results."

## Proposed Variation

As a result of the presented limitations in the existing four sets of equations, a fifth variation is proposed. This variation aims to ensure both the color volume and hue objectives, presented in the section titled "Variations Within the Standard," are accomplished. This variation is reached by adapting the YRGB method (Eqs. 5–7), so that:

**5. max(RGB)**

$$\boldsymbol{M_1 = \max(R_1,\ G_1,\ B_1)} \quad (9)$$

$$\boldsymbol{M_2} = \mathrm{EOTF}_{PQ}\left(\mathrm{EETF}\left(\mathrm{EOTF}_{PQ}^{-1}(\boldsymbol{M_1})\right)\right) \quad (10)$$

$$(R_2, G_2, B_2) = \frac{\boldsymbol{M_2}}{\boldsymbol{M_1}} \times (R_1, G_1, B_1). \quad (11)$$

Note: Bolded/blue font represents changes from the YRGB variation.

This variation ensures that the EETF is applied to the max light level of the three channels, per pixel, thereby constraining the colors to the desired 1,000 cd/m² RGB color volume and not a weighted 1,000 cd/m² luminance/luma/intensity channel. The maxRGB EETF mapping is illustrated conceptually in **Fig. 3**.

## Implemented Results

**Table 2** contains the results of the five implementations, as well as the 4,000 cd/m² source. The source had each variation of EETF applied to it with a target color volume of 1,000 cd/m².

Notice in **Table 2**, where $IC_TC_P$, $Y'C'_BC'_R$, and YRGB, all contain values exceeding the 1,000 cd/m² target (red). Meanwhile, the R'G'B' and maxRGB light levels are constrained to the 1,000 cd/m² limit that was imposed.

### *Plotted Representations of Color Difference*

The resulting RGB values from each of the implementations were then converted to Commission Internationale de l'Eclairage (CIE) XYZ values using the inverse of the matrix presented in ITU-R BT.2124-0[3] and reproduced in Eqs. 12 and 13.

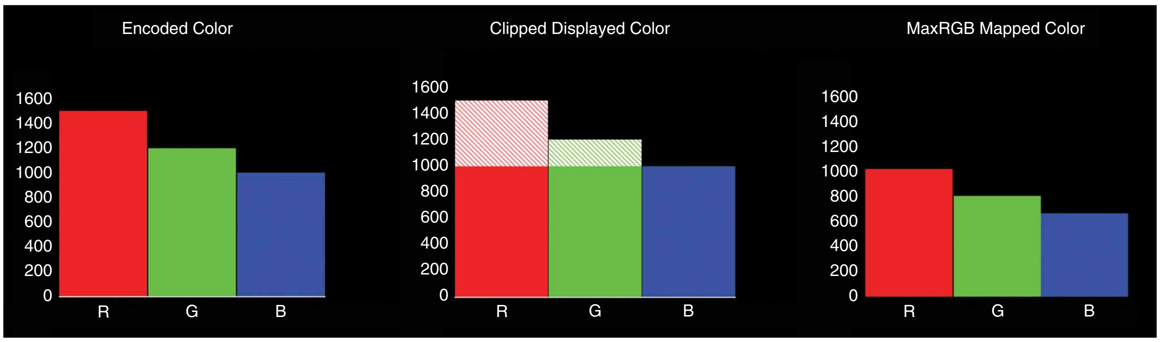


FIGURE 3. Conceptual example of maxRGB EETF mapping preserving the RGB ratios.

**TABLE 2. Calculations representing the effectiveness of each EETF implementation.**

| 4,000 cd/m² Source | | | | $IC_TC_P$—1,000 cd/m² Target | | | |
|---|---|---|---|---|---|---|---|
| Patch | R | G | B | Patch | R | G | B |
| Red | 3,009.9 | 182.92 | 0 | Red | 2,487.3 | 182.39 | 4.9712 |
| Green | 793 | 3,763.9 | 70.3 | Green | 395.71 | 1,409.86 | 65.069 |
| Blue | 189.92 | 49.826 | 3,929.4 | Blue | 194.95 | 51.7907 | 3,895.7 |
| **$Y'C'_BC'_R$ — 1,000 cd/m² Target** | | | | **YRGB — 1,000 cd/m² Target** | | | |
| Patch | R | G | B | Patch | R | G | B |
| Red | 3,009.9 | 182.92 | 0 | Red | 2,569.3 | 156.14 | 0 |
| Green | 351.58 | 1,969.3 | 20.498 | Green | 285.79 | 1,356.4 | 25.333 |
| Blue | 189.92 | 49.826 | 3,929.4 | Blue | 189.92 | 49.826 | 3,929.4 |
| **R′G′B′ — 1,000 cd/m² Target** | | | | **maxRGB — 1,000 cd/m² Target (Proposed)** | | | |
| Patch | R | G | B | Patch | R | G | B |
| Red | 998.32 | 182.92 | 0 | Red | 998.32 | 60.681 | 0 |
| Green | 721.46 | 1,000.00 | 70.3 | Green | 210.72 | 1,000.00 | 18.678 |
| Blue | 189.92 | 49.826 | 1,000.00 | Blue | 48.341 | 12.682 | 1,000.00 |

$$M_{XYZ\to RGB_{Rec.2020}} = \begin{bmatrix} 1.7167 & -0.3557 & -0.2534 \\ -0.667 & 1.6165 & 0.0158 \\ 0.0176 & -0.428 & 0.9421 \end{bmatrix} \quad (12)$$

$$M^{-1}_{XYZ\to RGB_{Rec.2020}} = M_{RGB_{Rec.2020}\to XYZ} \approx \begin{bmatrix} 0.6545 & 0.1898 & 0.1729 \\ 0.2690 & 0.6939 & 0.0607 \\ 0.1100 & 0.3117 & 1.0858 \end{bmatrix}. \quad (13)$$

These values were then converted to CIE 1976 chromaticity ($u'v'$) and plotted in **Fig. 4**. It is evident that there is a significant color difference in the R'G'B' implementation when compared to the original 4,000 cd/m² patch. It can also be seen that minimal hue changes can be achieved when leveraging the maxRGB variation.

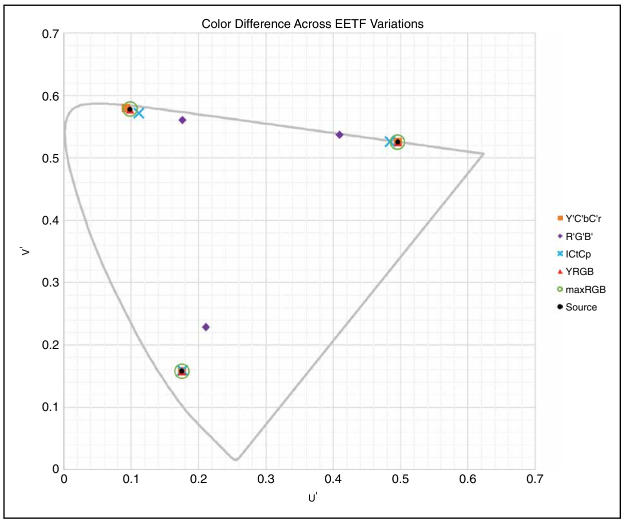


FIGURE 4. CIE 1976 chromaticity values before and after processing; using the four standardized EETF variations and the proposed fifth variation. Note: many of the values fall upon one another.

### *Qualitative Representation of Hue Change*

The conventional method of quantitative measurements for color difference is the use of a $\Delta E$-metric. However, since $\Delta E$-metrics contain a 3D Euclidean distance,[3] they do not accurately represent a change in color, independent of luminance. With the possibility of a drastic change in luminance (in this example 4,000 to 1,000 cd/m²),

**TABLE 3. Changes in hue angle between $C_T$ and $C_P$ within the $IC_TC_P$ color space.**

| | Blue patch | Green patch | Red patch | Sum(θ) |
|---|---|---|---|---|
| $IC_TC_P$ | 0.36° | 0.22° | 0.39° | 0.97° |
| $Y'C'_BC'_R$ | 0° | 0.55° | 0° | 0.55° |
| YRGB | 0° | 0.28° | 0.21° | 0.49° |
| R'G'B' | 22.85° | 11.76° | 12.98° | 47.59° |
| maxRGB | 1.12° | 0.38° | 1.53° | 3.03° |

**TABLE 4. Changes in hue angle between $u'$ and $v'$ within the $u'v'$ chromaticity.**

| | Blue patch | Green patch | Red patch | Sum(θ) |
|---|---|---|---|---|
| $IC_TC_P$ | 0.14° | 1.36° | 0.71° | 2.21° |
| $Y'C'_BC'_R$ | 0° | 0.63° | 0° | 0.63° |
| YRGB | 0° | 0° | 0° | 0° |
| R′G′B′ | 5.29° | 7.75° | 6.05° | 19.09° |
| maxRGB | 0° | 0° | 0° | 0° |

**TABLE 5. Standard/proposed EETF variations and objectives.**

| | $IC_TC_P$ | $Y'C'_BC'_R$ | YRGB | R'G'B' | maxRGB |
|---|---|---|---|---|---|
| Restricts To 1,000 cd/m$^2$ Color Volume | ✗ | ✗ | ✗ | ✓ | ✓ |
| Minimizes Difference in Hue | ✓ | ✓ | ✓ | ✗ | ✓ |

these metrics did not present meaningful data. Instead, changes in the hue angle (known to be the most noticeable parameter of color when changed) provided more meaningful data, while being independent of luminance.

The RGB data listed above were converted to the $IC_TC_P$ color space, as described by ITU-R BT.2100-2[4] and BT.2124-0[3]. These $C_TC_P$ data are plotted in **Fig. 5**. Like the *u'v'* plot, significant changes in hue are evident in the R'G'B' implementation compared to other variations of EETF.

Based on the coordinates of the $C_T$, $C_P$ values (plotted in **Fig. 5**), hue angles were derived for each channel for the five implementations. From here, the absolute differences in hue angles were calculated between the implementations and the original 4,000 cd/m$^2$ patch, as presented in **Table 3**. **Table 4** lists the hue-angle deviations using traditional *u'v'* chromaticity.

The R'G'B' implementation has a much higher difference in the hue angle than the remaining four variations. Although it does not achieve the lowest difference in the hue angle in ($C_T$, $C_P$), the maxRGB EETF has zero shift of the hue angle in traditional *u'v'* chromaticity, while also constraining the colors to the 1,000 cd/m$^2$ RGB color volume, as expressed at the top of this section.

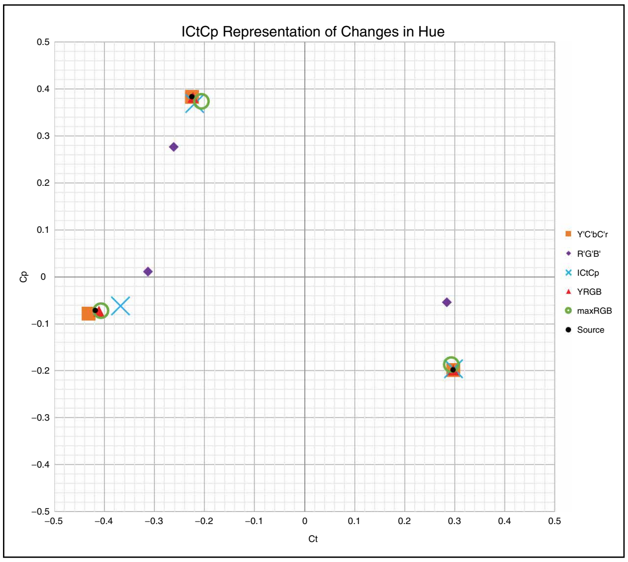


FIGURE 5. $C_T$, $C_P$ plot of the various implementations in the EETF.

## Conclusion

The maxRGB variation is the only implementation, out of the five presented in this article and summarized in **Table 5**, that accomplishes both objectives. The variation restricts the color volume of an output code value to 1,000 cd/m$^2$, constraining it to the 1,000 cd/m$^2$ RGB color volume. The maxRGB implementation also reduces the difference in the hue angle, thereby minimizing noticeable changes in color. It should be noted that, although 4,000 cd/m$^2$ being mapped to 1,000 cd/m$^2$ is used throughout this article as an example, any max luminance can be mapped to a lower luminance. Variation in performance may, however, vary.

## References

1. International Telecommunications Union—Radiocommunication (ITU-R), Recommendation BT.2390-7, “High Dynamic Range Television for Production and International Programme Exchange,” Jul. 2019.

2. SMPTE, ST 2113:2018, “Colorimetry of P3 Color Spaces,” 2018.

3. International Telecommunications Union—Radiocommunication (ITU-R), Recommendation BT.2124-0, “Objective Metric for the Assessment of the Potential Visibility of Colour Differences in Television,” Jan. 2019.

4. International Telecommunications Union—Radiocommunication (ITU-R), Recommendation BT.2100-2, “Image Parameter Values for High Dynamic Range Television for Use in Production and International Programme Exchange,” Jul. 2018.

## About the Authors

***Adam Burke*** is a recent graduate of the Motion Picture Science program at Rochester Institute of Technology (RIT), Rochester, NY. In the summer of 2019, he interned with the Warner Bros., Emerging Technology Division, Burbank, CA, where he worked on a variety of image quality and color science projects, one of which was the basis for this article. He received the 2019 Louis F. Wolf Jr. Memorial Scholarship from SMPTE and has acted as the vice president of the RIT Student SMPTE chapter during his senior year. Throughout his college career, he worked for RIT SportsZone, the university’s sports broadcasting group, as a student director. In addition, he managed the technical aspects of the end-of-semester, academic screenings for the School of Film and Animation. He now looks forward to pursuing a career where creativity and innovation are at the forefront of the work.

***Michael D. Smith*** has been working as a digital imaging and intellectual property consultant since 2003. He is currently a co-editor of the JPEG 2000 High-throughput image compression standard. In 2018, he received a screen credit for his work on *Mary Poppins Returns*, which was related to the color science involved in integrating a traditional 2D animation workflow with a modern Academy Color Encoding System (ACES) production pipeline. From 2013 to 2015, he was the co-chair of Blu-ray Disc Association's ultrahigh-definition (UHD)-Task Force (TF) video subgroup, which defined the video-related requirements for the UHD Blu-ray disc format. His work on more than 35 intellectual property matters related to infringement and validity of patents has resulted in payments of approximately $1.7 billion. He was an editor of the book *3-D Cinema and Television Technology: The First 100 Years*, published by SMPTE in 2011. He received BS and MS degrees in electrical engineering from the University of California, Los Angeles, CA, in 2001 and 2004, respectively.

***Michael Zink*** is the vice president of technology at Warner Bros. (WB), Burbank, CA, where he is responsible for exploring emerging technologies to enhance WB's capabilities for production, post-production, and distribution. This includes assessing new technologies and assisting with the setup and integration of digital workflows. He also participates in a number of standards associations such as the Consumer Technology Association (CTA), Digital Cinema Initiatives (DCI), and SMPTE, among others and also serves as the chairman for the Ultra High Definition Alliance (UHDA). Prior to joining WB, in 2014, he worked at Technicolor in London, UK and Los Angeles, CA, for more than ten years, most recently as the vice president of technology strategy, where he was responsible for launching the production efforts around various new optical disc formats. Additionally, he was responsible for the promotion and adoption of Technicolor technology solutions within industry groups. Earlier in his career, he worked for several media production facilities in Germany.